

(i) Printed Pages: 3

Roll No.

(ii) Questions : 8

Sub. Code :

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Exam. Code :

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B.A./B.Sc. (General) 3rd Semester

(1129)

MATHEMATICS

Paper : (I. Advanced Calculus-I)

Time Allowed : Three Hours]

[Maximum Marks : 30

Note :— Attempt **FIVE** questions in all selecting at least **TWO** questions each Section. All questions carry equal marks.

SECTION—A

1. (a) By using definition, prove that :

$$\lim_{(x,y) \rightarrow (1,2)} x^2 + 5y = 11.$$

(b) Show that the function defined by :

$$f(x, y) = \begin{cases} \frac{x^3 - y^3}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

is continuous at (0, 0).

3,3

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[Turn over

2. (a) If $V = r^m$ where $r = \sqrt{x^2 + y^2 + z^2}$, show that

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = m(m+1)r^{m-2}$$

- (b) Discuss the differentiability of the function $f(x, y) = (xy)^{1/3}$ at $(0, 0)$. 3,3

3. (a) Let $f(x, y) = \frac{xy(x-y)}{x+y}$ ($x, y \neq (0, 0)$) and $f(0, 0) = 0$

show that $f_{xy}(0, 0) \neq f_{yx}(0, 0)$.

- (b) If $\vec{r} = t^3 \hat{i} + \left(2t^3 - \frac{1}{5t^2}\right) \hat{j}$

show that $\vec{r} \times \frac{d\vec{r}}{dt} = \vec{k}$. 3,3

4. (a) In what direction from $(3, 1, -2)$ is the directional derivative of $\phi = x^2 y^2 z^4$ maximum? Find the magnitude of this maximum.

- (b) Show that :

$$\nabla^2 \left(\frac{x}{r^3} \right) = 0. \quad 3,3$$

SECTION—B

5. (a) If $u = \sin^{-1} \frac{x+y}{\sqrt{x} + \sqrt{y}}$, prove that :

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{-\sin u \cos 2u}{4 \cos^3 u}$$

- (b) Expand $x^4 + x^2 y^2 - y^4$ in the neighbourhood of the point $(1, 1)$ upto the terms of second degree. 3,3

6. (a) If u, v, w be the roots of equation

$$(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0.$$

Prove that :

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = \frac{-2(x - y)(y - z)(z - x)}{(u - v)(v - w)(w - u)}.$$

- (b) Show that the functions $u = x^2 + y^2 + z^2$, $v = xy - xz - yz$, $w = x + y - z$ are dependent and find a relation connecting them. 3,3
7. (a) Find the envelope of the circles passing through origin and whose centre lies on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.
- (b) Find the evolute of the parabola $y^2 = 4ax$. 3,3
8. (a) Prove that $u = x^2y^2 - 5x^2 - 8xy - 5y^2$ is maximum at $x = y = 0$.
- (b) Prove that of all rectangular parallelopipeds of the same volume, the cube has the least surface. 3,3

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